[11:00am-11:15am] Resources for midterm #1

Over a dozen previous exams are available on the course website:

https://users.ece.utexas.edu/~bevans/courses/signals/lectures/midterm1/index.html

The exam is open book, open laptop, and open notes. However, all networking must be disabled and AI tools (including local AI tools) are not allowed.

[11:15-11:25] Review: sampling theorem

$$x(t) = \cos(2\pi f_0 t), \quad -\infty < t < \infty$$

Recall that the Fourier series representation of a cosine can be determined using the inverse Euler identity:

$$\cos(2\pi f_0 t) = \frac{1}{2} e^{-j2\pi f_0 t} + \frac{1}{2} e^{j2\pi f_0 t}$$

Thus, exactly two frequencies are present: $-f_0$ and f_0 . Both have a Fourier series coefficient of $\frac{1}{2}$.

To reconstruct a signal from its samples, the sampling theorem requires $f_s > 2f_{max}$. In this case, $f_s > 2f_0$. However, the sampling theorem only tells us that reconstruction is possible; it does instruct how to reconstruct the signal (interpolation.)

[11:25-11:45] Continuous → discrete conversion (sampling)

The process of converting a signal from continuous time to discrete time is also called sampling. During the sampling process, we can only capture frequencies up to $f_s/2$. Any frequencies above this will alias. An analog lowpass filter with cutoff frequency near $f_s/2$ applied before sampling will prevent aliasing.

An ideal lowpass filter will only pass frequencies between $-f_s/2 < f < f_s/2$. In other words, its frequency response is a rectangular pulse. However, it's not possible to build this ideal lowpass filter.

In practice, the lowpass filter cannot immediately transition from the passband to the stopband. Instead, a transition band (roughly 10% of the filter bandwidth) is necessary. Frequencies in the transition band will be partially attenuated.

Audio anti-aliasing example: Only frequencies below 20 kHz are audible, so we can apply a lowpass anti-aliasing filter that attenuates higher frequencies. The transition band extends from 20 kHz to 22 kHz (about 10%), and the audio signal is sampled at 44 kHz. An RC circuit can be used as the lowpass filter (see handout O).

[11:55-] Folding

$$x_1[n] = \cos(0.4\pi n), \quad x_2[n] = \cos(2.4\pi n)$$

If $f_s = 1$ Hz, then we capture frequencies between -0.5Hz < f < 0.5 Hz.

However, $x_2[n]$ contains frequencies $\pm f_0 = \pm 1.2$ Hz, so it will alias.

$$0.4\pi = \frac{2\pi f_{alias}}{f_s} \rightarrow f_{alias} = 0.2 \ Hz$$

More generally if we sample a sinusoid at $T_s = 1/f_s$,

$$x(t) = \cos(2\pi f_0 t + \phi)$$

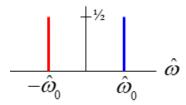
$$y(t) = \cos(2\pi (f_0 + \ell f_s)t + \phi)$$

$$x[n] = x(nT_s) = A\cos(2\pi f_0 T_s n + \phi) = A\cos\left(2\pi \frac{f_0}{f_s} n + \phi\right)$$

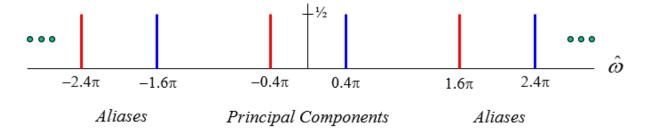
$$y[n] = y(nT_s) = A\cos\left(2\pi \frac{f_0}{f_s} n + \phi\right)$$

So x[n] is indistinguishable from y[n]. In other words, once aliasing has occurred, there are an infinite number of frequencies that could explain the observed signal.

$$\cos(\widehat{\omega}n) = 0.5 \left(e^{-j\widehat{\omega}n} + e^{j\widehat{\omega}n} \right)$$
 (frequency components at $-\widehat{\omega}$ and $\widehat{\omega}$)



After sampling, frequency components will also occur at multiples of f_s

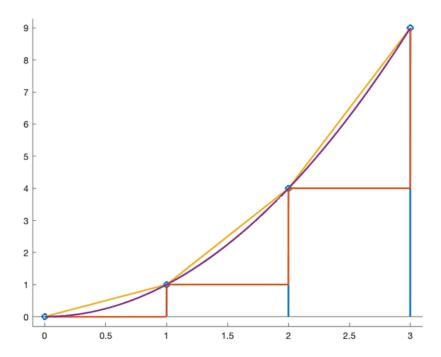


[12:10-] Discrete-to-continuous conversion

Several ways to fill in the values between samples (interpolation):

- Zero-order hold (stair steps)
- Linear interpolation (draw straight line between samples)
- Curve fitting (e.g. cubic polynomial)

x	f(x)
0	0.0
1	1.0
2	4.0
3	9.0



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Sampling at $f_s = 1$ Hz

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f5=1H2

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