[11:00am-11:15am] Resources for midterm #1

Over a dozen previous exams are available on the course website:

https://users.ece.utexas.edu/~bevans/courses/signals/lectures/midterm1/index.html

The exam is open book, open laptop, and open notes. However, all networking must be disabled and AI tools (including local AI tools) are not allowed.

[11:15-11:25] Review: sampling theorem

$$x(t) = \cos(2\pi f_0 t)\,, \qquad -\infty < t < \infty$$

Recall that the Fourier series representation of a cosine can be determined using the inverse Euler identity:

$$\cos(2\pi f_0 t) = \frac{1}{2}e^{-j2\pi f_0 t} + \frac{1}{2}e^{j2\pi f_0 t}$$

Thus, exactly two frequencies are present: $-f_0$ and f_0. Both have a Fourier series coefficient of ½.

To reconstruct a signal from its samples, the sampling theorem requires $f_s > 2f_{max}$. In this case, $f_s > 2f_0$. However, the sampling theorem only tells us that reconstruction is possible; it does instruct how to reconstruct the signal (interpolation.)

[11:25-11:45] Continuous → discrete conversion (sampling)

The process of converting a signal from continuous time to discrete time is also called sampling. During the sampling process, we can only capture frequencies up to $f_s/2$. Any frequencies above this will alias. An analog lowpass filter with cutoff frequency near $f_s/2$ applied before sampling will prevent aliasing.

An ideal lowpass filter will only pass frequencies between $-f_s/2 < f < f_s/2$. In other words, its frequency response is a rectangular pulse. However, it's not possible to build this ideal lowpass filter.

In practice, the lowpass filter cannot immediately transition from the passband to the stopband. Instead, a transition band (roughly 10% of the filter bandwidth) is necessary. Frequencies in the transition band will be partially attenuated.

Audio anti-aliasing example: Only frequencies below 20 kHz are audible, so we can apply a lowpass anti-aliasing filter that attenuates higher frequencies. The transition band extends from 20 kHz to 22 kHz (about 10%), and the audio signal is sampled at 44 kHz. An RC circuit can be used as the lowpass filter (see handout O).

[11:55-] Folding

$$x_1[n] = \cos(0.4\pi n)\,, \qquad x_2[n] = \cos(2.4\pi n)$$

If $f_s = 1$ Hz, then we capture frequencies between $-0.5Hz < f < 0.5\ Hz$.

However, $x_2[n]$ contains frequencies $\pm f_0 = \pm 1.2$ Hz, so it will alias.

$$0.4\pi = \frac{2\pi f_{alias}}{f_s} \rightarrow f_{alias} = 0.2\ Hz$$

More generally if we sample a sinusoid at $T_s = 1/f_s$,

$$x(t) = \cos(2\pi f_0 t + \phi)$$

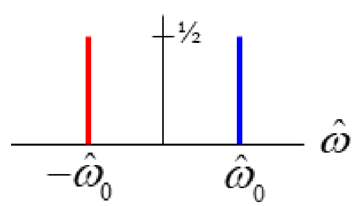
$$y(t) = \cos(2\pi(f_0 + \ell f_s)t + \phi)$$

$$x[n] = x(nT_s) = A\cos(2\pi f_0 T_s n + \phi) = A\cos\left(2\pi \frac{f_0}{f_s} n + \phi\right)$$

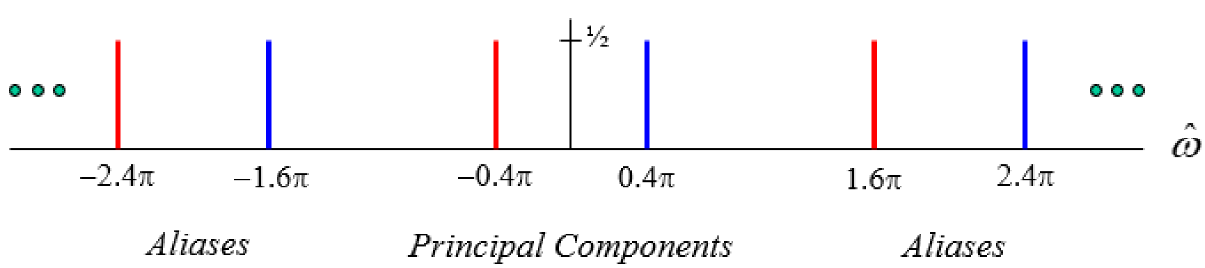
$$y[n] = y(nT_s) = A\cos\left(2\pi \frac{f_0}{f_s} n + \phi\right)$$

So $x[n]$ is indistinguishable from $y[n]$. In other words, once aliasing has occurred, there are an infinite number of frequencies that could explain the observed signal.

$$\cos(\hat{\omega} n) = 0.5\left(e^{-j\hat{\omega} n} + e^{j\hat{\omega} n}\right) \qquad \text{(frequency components at } -\hat{\omega} \text{ and } \hat{\omega}\text{)}$$



After sampling, frequency components will also occur at multiples of f_s

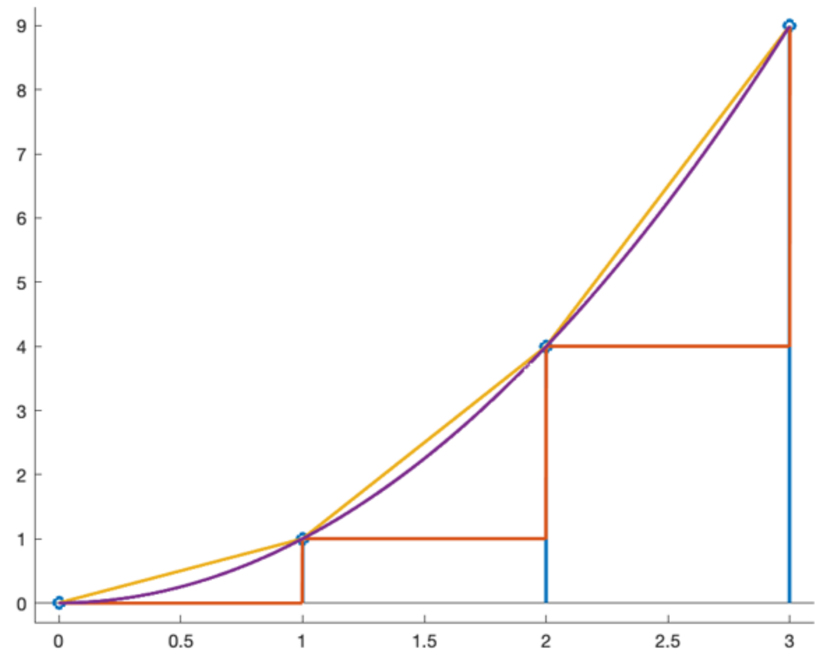


[12:10-] Discrete-to-continuous conversion

Several ways to fill in the values between samples (interpolation):

- **Zero-order hold (stair steps)**
- **Linear interpolation (draw straight line between samples)**
- **Curve fitting (e.g. cubic polynomial)**

x	$f(x)$
0	0.0
1	1.0
2	4.0
3	9.0



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Sampling at $f_s = 1\ Hz$

- capture frequencies $-0.5\ Hz < f < 0.5\ Hz$

$f_0 = 1.2\ Hz \rightarrow \hat{\omega}_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{1.2\ Hz}{1\ Hz} = 2.4\pi$

$\cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n) = \cos(\underbrace{0.4}\pi n)$

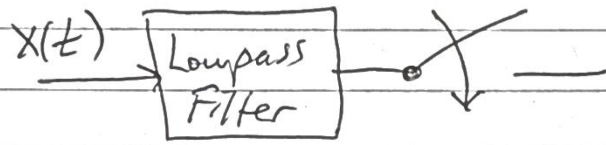
aliased freq.

$0.4\pi = 2\pi \frac{f_{alias}}{f_s} = 2\pi f_{alias} \rightarrow f_{alias} = 0.2\ Hz$

$\uparrow f_s = 1\ Hz$

Sampling in practice

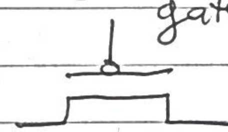
Slide 5-3. Review



attenuate frequencies outside the range

$-\frac{1}{2} f_s < f < \frac{1}{2} f_s$

switch that closes and opens every T_s second



pass transistor

$f_{stop} = 22$ KHz

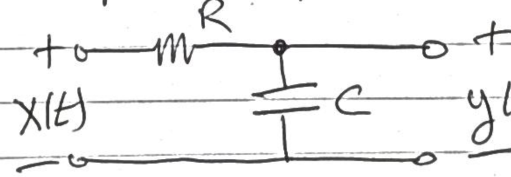
$f_s > 2 f_{stop}$

$f_{stop} = 10\% f_{max} + f_{max}$

Ideal Lowpass Filter (rectangular pulse)

$-f_{max}$ $f_{max} = 20$ kHz for audio for human hearing

Example Lowpass Filter



Handout O and more in ECE 319K Intro to Embedded